Parameters and states estimation for Dengue epidemic model

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***Abstract*— This paper focuses on the states estimation and parameter identification for nonlinear Dengue epidemic model. This model supplies a change of coordinates that transforms the studied model into an extended output depending nonlinear observer normal form. Then we can use the high gain observer strategy, which allows both the parameter identification and the state estimation.**

I. INTRODUCTION

The Dengue fever is endemic throughout the tropics and no specific treatment has been developed yet. The disease is transmitted between humans throughout mosquito bites. This spreading process through mosquitoes is given by a nonlinear dynamical system by involving two species (mosquitoes and humans). Each species is partitioned into two populations; the susceptible and the infected one. In this process, the mosquitoes are considered as vectors of disease. In order to minimize its impact on society, numerous studies have been conducted to understand the behavior of dynamics of such diseases which resulted in the development of several models aimed to analyze and predict the evolution of such epidemic in the purpose of minimizing its impact on society.

The modeling of epidemic spread was initiated by [1], that had led to the well-known SIR model (Susceptible, Infected and Removed). Afterwards, it has been improved and adapted according to the type of disease. Based on these models, several works are realized to study the volatility of diseases [2], and to develop vaccination strategy [3]. However, as far as we know, there are few works dedicated to the observer design [4].

To achieve an observer design for the dengue epidemic model, we will use the approach based on nonlinear observer normal forms (NONF). This concept has been set up in [5] for a varying time and in [6] for an invariant-time single output nonlinear dynamical system. Then, it has been adopted for the multiple outputs case in [7], [8]. Afterwards, several algorithms have been developed in [9], [10], [11], [12], [13], [14], [15] and [16]. However, the conditions for the existence of diffeomorphism reported in the above works are relatively too restrictive. Therefore, to enhance this concept and to expand the class of nonlinear systems that allow a change of coordinates, another interesting form is the so-called depending output observer normal form introduced by [17], and then improved in [18], [19], [20], [21], [22] and [23]. The most recent normal form, called extended nonlinear observer normal form due to [24], allows to enlarge the class of observer normal forms. It has been generalized by [25], [26], [27], [28] and [29]. The idea of the last form is to add an auxiliary dynamics into the original system such that the augmented system fulfills commutativity condition of Krener and Isidori frame [6], which guarantees a transformation of the studied system into an extended normal form [30], [31]. On the other hand, this observer normal form provides an adaptive observer to resolve the simultaneous parameters identification and state estimation problem [32]. Thus, we believe that its application to epidemics model can constitute a very powerful decision-making tool in the vaccination campaign and fighting against diseases. The objective of this paper is intended to show the effectiveness of the observer design to estimate the spreading disease and to identify the parameters in the epidemic model.

This paper is organized as follows. Section II illustrates the formalization of extended output depending observer normal form. Section III presents the dengue epidemic model. Section IV supplies a change of coordinates allowing the transformation of the dengue epidemic model into an extended output depending normal form. Section V deals with simultaneous state estimation and parameter identification of the studied system.

II. BACKGROUND ON NONLINEAR EXTENDED OBSERVER NORMAL FORMS

We Consider a nonlinear system in the following form:

$$\left\{\begin{array}{l}\dot{x}=f(x)\\ y=h(x)\end{array}\right. \tag{1}$$

where $x \in U \subseteq \mathbb{R}^n$ represents the state and $y \in \mathbb{R}^p$ denotes the output. We assume that the vector field f and the output function h are sufficiently smooth. In the following, we also assume that the pair (h, f) satisfies the observability rank condition, thus the 1-forms $\theta_i = dL_f^{i-1}h$ for $i = 1 : n$ are linearly independent where $L_f^{i-1}h$ is the $(i-1)^{th}$ Lie derivative of the output h in the direction of the vector field f, and d is the usual differential. Under this assumption, theoretically the state can be estimated as $x = \sigma(y, \dot{y}, ..., y^{n-1})$. However, it has been established that the use of successive output derivatives amplifies the noise in the measurement. Therefore, the observer design is a powerful tool to address the state estimation problem. To do so, we firstly present a class of nonlinear systems for which the design of observer is relatively straightforward.

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Consider the nonlinear system (1), we seek an auxiliary dynamics $\dot{w} = \eta(y,w)$ so that the following extended dynamical system:

$$\dot{x} = f(x) \quad (2)$$
$$\dot{w} = \eta(y,w) \quad (3)$$
$$y = h(x) \quad (4)$$

can be transformed through a change of coordinates $(\xi^T,\zeta)^T = \phi(x,w)$ into the following extended output depending observer normal form:

$$\dot{\xi} = A(y,w)\xi + B(y,w) \quad (5)$$
$$\dot{\zeta} = B_{n+1}(y,w) \quad (6)$$
$$y = C\xi \quad (7)$$

where $\xi \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $\zeta \in \mathbb{R}$, $w \in \mathbb{R}$ is an extra-output, and

$$A(y,w) = \begin{pmatrix} 0 & ... & ... & ... & 0 \\ \alpha_2(y,w) & 0 & ... & ... & 0 \\ 0 & \alpha_3(y,w) & ... & ... & ... \\ 0 & ... & ... & ... & ... \\ 0 & ... & ... & \alpha_n(y,w) & 0 \end{pmatrix} \quad (8)$$

where $\alpha_i(y,w) \neq 0$ for $i = 2:n$ is a function depending only on the output y and the extra-output w. The normal form (5-7) has a great interest because it supports Kalman-like observer or the following high gain observer presented in [33]:

$$\dot{\hat{\xi}} = A(y,w)\hat{\xi} + B(w,y) - \Gamma^{-1}(y)R_\rho^{-1}C^T(C\hat{\xi} - \overline{y}) \quad (9)$$
$$0 = \rho R_\rho + G^T R_\rho + R_\rho G - C^T C \quad (10)$$

where

$$G = \begin{pmatrix} 0 & \cdots & 0 & 0 \\ 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{pmatrix}$$

$$\Gamma(y,w) = diag[\prod_{i=2}^{n}\alpha_i(y,w), \prod_{i=3}^{n}\alpha_i(y,w), \cdots, \alpha_n(y,w), 1]$$

$$R_\rho(n+1-i, n+1-j) = \frac{(-1)^{i+j}C_{i+j-2}^{j-1}}{\rho^{i+j-1}}$$

for $1 \leq i \leq n$ and $1 \leq j \leq n$.
If we set $e = \hat{\xi} - \xi$ to be the observation error, then we see that its error dynamics is governed by the following equation:

$$\dot{e} = \dot{\hat{\xi}} - \dot{\xi} = (A(y,w) - \Gamma^{-1}(y,w)R_\rho^{-1}C^TC)e.$$

If y and w are bounded, then the observation error dynamics is exponentially stable by well choosing ρ.

III. DENGUE DISEASE MODEL

Dengue is a viral infection transmitted by the bite of mosquito Aedes aegypti to human. It is an endemic in many countries through the world, it has not a specifical treatment and the more adapted solution is the prevention.

Throughout this paper, we consider the dengue epidemic model extracted from [34], [35], whose behavior is governed by a set of differential equations describing epidemic transmission between humans and mosquitoes:

$$\begin{cases} \frac{dS(t)}{dt} = b_1 - \lambda_1 SV - \mu_1 S \\ \frac{dI(t)}{dt} = \lambda_1 SV - \gamma I - \mu_1 I \\ \frac{dM(t)}{dt} = b_2 - \lambda_2 MI - \mu_2 M \\ \frac{dV(t)}{dt} = \lambda_2 MI - \mu_2 V \end{cases} \quad (11)$$

where $S(t)$ and $I(t)$ are respectively the susceptible and infected individuals in human population, $M(t)$ and $V(t)$ are respectively the susceptible and infected mosquitoes. The involving parameters are the following senses: b_1 is the rate of natural birth on the host population, μ_1 is the natural death rate related to host population, λ_1 is the rate daily biting (or the rate of transmission from V), γ is the rate at which the infected individuals are recovered, b_2 is the birth rate of vector population (i-e mosquitoes), λ_2 is the transmission rate from I to vectors (or rate daily biting of I), and μ_2 is the natural death rate of mosquitoes (i-e vectors).

Throughout this paper, it is assumed that all information concerning humans data $S(t)$ and $I(t)$ will be available thanks to the health department. Therefore, they are taken as the outputs of the model:

$$\begin{cases} h_1 = y_1 = S \\ h_2 = y_1 = I \end{cases} \quad (12)$$

Now, we will check the observability rank condition of the above dynamical system (11) with the output defined in (12). For this, let us compute the Lie derivative of the outputs as follows:

$$\begin{aligned} \theta_1 &= dL_f^0 h_1 = dS \\ \theta_2 &= dL_f^1 h_1 = (\lambda_1 V - \mu_1)\, dS + \lambda_1 S dV \\ \theta_3 &= dL_f^2 h_1 \\ &= -\left(\lambda_1 b_1 - \lambda_1\mu_2 S - 2\mu_1\lambda_1 S - 2\lambda_1^2 SV\right) dV \\ &\quad + \left(\mu_1^2 + \lambda_1(2\mu_1 + \mu_2)V + \lambda_1^2 V^2 - \lambda_1\lambda_2 MI\right) dS \\ &\quad - \lambda_1\lambda_2 SI dM - \lambda_1\lambda_2 SM dI \\ \theta_4 &= dh_2 = dI \end{aligned}$$

We see that θ_i for $1 \leq i \leq 4$ are linearly independent which implies the observability of system (11). Thus, we can estimate the state $V(t)$ and $M(t)$ from the outputs and their derivatives i.e. $(V,M) = \sigma(y, \dot{y}, y^{(2)})$.

In order to simplify the calculation, as $I(t)$ and $V(t)$ are measured, then we can remove the dynamics of I in (11). Specifically, we will deal with the following reduced dynamical system:

$$\begin{cases} \frac{dS(t)}{dt} = b_1 - \lambda_1 SV - \mu_1 S \\ \frac{dM(t)}{dt} = b_2 - \lambda_2 MI - \mu_2 M \\ \frac{dV(t)}{dt} = \lambda_2 MI - \mu_2 V \end{cases} \quad (13)$$

IV. EXTENDED OUTPUT DEPENDING OBSERVER NORMAL FORM

This section provides a change of coordinates that transforms the model (13) into (5-7). For this we will process in two steps.

A. First change of coordinates

This subsection deals with a first transformation that enables to transform the dynamical system (13) into another dynamical system with depending output linear part modulo nonlinear terms which are only function of outputs I and S which will allow, thereafter, an easy use of the extended transformation algorithm.

Proposition 1: The following change of coordinates

$$z_3 = -\frac{1}{\lambda_1}\ln(S) \tag{14}$$

$$z_2 = \frac{1}{\lambda_2}V - \frac{1}{\lambda_1}\left(\frac{\mu_2}{\lambda_2} + S + I\right)\ln(S) + \frac{S}{\lambda_1} \tag{15}$$

$$z_1 = M + V \tag{16}$$

transforms (13) into the following dynamical system

$$\begin{cases} \dot{z}_1 = b_2 - \mu_2 z_1 \\ \dot{z}_2 = I z_1 + \beta_2(y) \\ \dot{z}_3 = \lambda_2 z_2 + \beta_3(y) \\ \widetilde{y} = z_3 \end{cases} \tag{17}$$

where

$$\begin{aligned} \beta_3(y) &= -\lambda_2\left(-\frac{\ln(S)}{\lambda_1}\left(\frac{\mu_2}{\lambda_2} + S + I\right) + \frac{S}{\lambda_1}\right) \\ &\quad -\frac{b_1}{\lambda_1 S} + \frac{\mu_1}{\lambda_1} \\ \beta_2(y) &= \left(-\frac{b_1}{\lambda_1 S} + \frac{\mu_1}{\lambda_1}\right)\left(S + I + \frac{\mu_2}{\lambda_2}\right) \\ &\quad -\frac{\ln(S)}{\lambda_1}\left(b_1 - \mu_1 S - (\gamma + \mu_1) I\right) \\ &\quad +\frac{(b_1 - \mu_1 S)}{\lambda_1} \end{aligned}$$

Proof: A direct derivation of change of coordinates (16) implies $\dot{z}_3 = -\frac{\dot{S}}{\lambda_1 S}$. Now, from the model (11), we obtain:

$$\begin{aligned} \dot{z}_3 &= \lambda_2 z_2 - \lambda_2\left(-\frac{\ln(S)}{\lambda_1}\left(\frac{\mu_2}{\lambda_2} + S + I\right) + \frac{S}{\lambda_1}\right) \\ &\quad -\frac{b_1}{\lambda_1 S} + \frac{\mu_1}{\lambda_1} \end{aligned}$$

which can be rewritten into the following compact form:

$$\dot{z}_3 = \lambda_2 z_2 + \beta_3(y).$$

In the same way, the derivative of the second component of change of coordinates (15) gives:

$$\begin{aligned} \dot{z}_2 &= \frac{1}{\lambda_2}\dot{V} + \mu_2\dot{z}_3 + \dot{z}_3(S + I) + z_3\left(\dot{S} + \dot{I}\right) + \frac{1}{\lambda_1}\dot{S} \\ &= I z_1 + \left(-\frac{b_1}{\lambda_1 S} + \frac{\mu_1}{\lambda_1}\right)\left(S + I + \frac{\mu_2}{\lambda_2}\right) \\ &\quad -\frac{\ln(S)}{\lambda_1}\left(b_1 - \mu_1 S - (\gamma + \mu_1) I\right) + \frac{(b_1 - \mu_1 S)}{\lambda_1} \end{aligned}$$

Therefore, we obtain the second differential equation in Proposition 1 as follows:

$$\dot{z}_2 = I z_1 + \beta_2(y).$$

For the third component of the change of coordinates (16), a straightforward calculation leads to

$$\dot{z}_1 = \dot{M} + \dot{V} = b_2 - \mu_2 z_1$$

■

However, due to the first dynamics in system (17), one can see that it is different from the proposed observer normal form (5)-(7).And this motivates us to introduce the concept of extended dynamics (or immersion) to overcome this limitation.

B. Extended Dynamics

In this subsection, we will use the concept of extended dynamics. For this, let us extend the dynamical system (17) as follows

$$\begin{cases} \dot{z}_1 = b_2 - \mu_2 z_1 \\ \dot{z}_2 = I z_1 + \beta_2(y) \\ \dot{z}_3 = \lambda_2 z_2 + \beta_3(y) \\ \dot{w} = \eta(y, w) \end{cases} \tag{18}$$

where $w \in \mathbb{R}$ is an auxiliary variable considered as an extra output and $\eta(y, w)$ is a function to be determined (for more details see [36], [24], [26], [27] and [28]).

Proposition 2: The following change of coordinates

$$\xi_1 = l(w) z_1 \tag{19}$$

$$\xi_2 = z_2, \quad \xi_3 = z_3, \quad \xi_4 = w \tag{20}$$

transforms the dynamical system (18) into the following extended nonlinear output depending observer normal form

$$\begin{cases} \dot{\xi}_1 = B_1(y, w) \\ \dot{\xi}_2 = \frac{I}{l(w)}\xi_1 + B_2(y, w) \\ \dot{\xi}_3 = \lambda_2\xi_2 + B_3(y, w) \\ \dot{\xi}_4 = \eta(y, w) \\ y = S \end{cases} \tag{21}$$

where $\eta(w)$ is only function of w and $l(w) = e^{\int_0^w \frac{\mu_2}{\eta(s)}}$.

Proof: Firstly, for a given $\eta(y, w) \neq 0$, let us firstly seek the function $l(w)$. A direct derivative of the change of coordinates (19) implies

$$\dot{\xi}_1 = z_1 l'(w)\dot{w} + l(w)\dot{z}_1.$$

Now, from the first and the fourth differential equations of (18), we obtain

$$\dot{\xi}_1 = z_1 l'(w)\eta(y, w) - l(w)\mu_2 z_1 + l(w) b_2.$$

As intended by observer normal form (21), the dynamics $\dot{\xi}_1$ should be a function of the output (S, I, w), therefore, we require that

$$z_1 l'(w)\eta(y, w) - l(w)\mu_2 z_1 = 0$$

which leads to the following differential equation

$$\frac{l'(w)}{l(w)} = \frac{\mu_2}{\eta(y, w)}. \tag{22}$$

As $l(w)$ is chosen as only a function of w, then from the last equation (22), the function $\eta(w, y) = \eta(w)$ must be also only a function of w.

Then, by integrating of equation (22) we obtain

$$l(w) = e^{\int_0^w \frac{\mu_2}{\eta(s)} ds}.$$

Therefore, the dynamics of ξ_1 becomes

$$\dot{\xi}_1 = B_1(y, w) = l(w) b_2.$$

Now, taking into account the dynamical system (18), the derivative of the second variable (20) implies:

$$\begin{aligned}\dot{\xi}_2 &= \frac{I}{l(w)}\xi_1 + \left(-\frac{b_1}{\lambda_1 S} + \frac{\mu_1}{\lambda_1}\right)\left(S + I + \frac{\mu_2}{\lambda_2}\right) \\ &\quad -\frac{1}{\lambda_1}\left(b_1 - \mu_1 S - (\gamma + \mu_1) I\right)(\ln(S)) \\ &\quad +\frac{1}{\lambda_1}\left(b_1 - \mu_1 S\right)\end{aligned}$$

which can be written as follows:

$$\dot{\xi}_2 = \alpha_2(y, w)\,\xi_1 + B_2(y, w).$$

In the same way, the derivative of ξ_3 leads to

$$\begin{aligned}\dot{\xi}_3 &= \lambda_2\xi_2 - \lambda_2\left(-\frac{1}{\lambda_1}\ln(S)\left(\frac{\mu_2}{\lambda_2} + S + I\right) + \frac{1}{\lambda_1}S\right) \\ &\quad -\frac{b_1}{\lambda_1 S} + \frac{\mu_1}{\lambda_1}\end{aligned}$$

and this can be described into the following compact form

$$\dot{\xi}_3 = \alpha_3(y, w)\,\xi_2 + B_2(y, w)$$

and finally the dynamics of ξ_4 is given by

$$\dot{\xi}_4 = \eta(y, w) = B_4(y, w).$$

■

C. Simulation results

For simulation, we use the following parameters: $b1 = 0.01$, $\lambda_1 = 5E-4$, $\mu_1 = 0.01$, $\gamma = 0.01$, $\lambda_2 = 3E-4 : 10$, $b_2 = 60$; $\mu_2 = 0.5$. The initial conditions are $S(0) = 784$, $I(0) = 216$, $R(0) = 0$; $M(0) = 0$, $V(0) = 100$. To ensure the boundedness of w, we choose $\dot{w} = \eta(w) = \frac{\sin aw}{aw}\mu_2$ with $a = 0.0005$. The simulation results are presented in Fig (1-3)

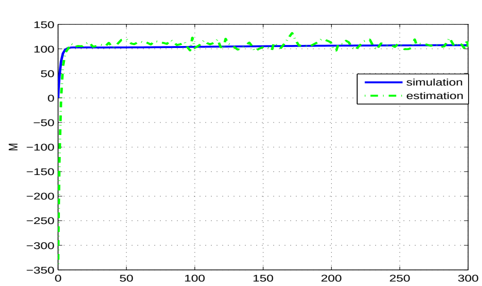

Fig. 1. Evolution of population (M)

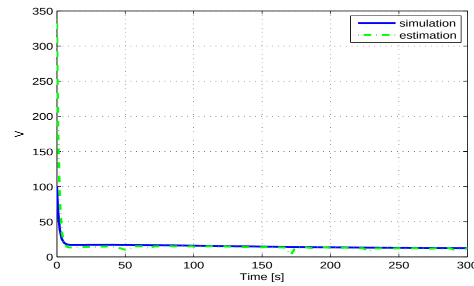

Fig. 2. Evolution of population (V)

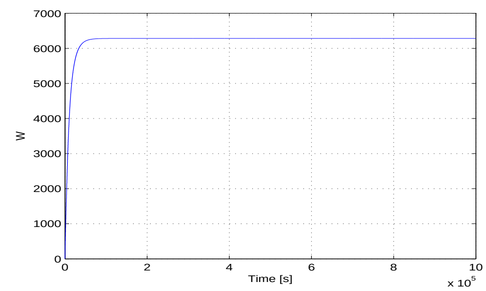

Fig. 3. Boundedness of auxiliary dynamics (w)

V. ADAPTIVE OBSERVER

Unlike the above section, this section deals with simultaneous parameter identification and state estimation for system (13). It assumes that the parameters b_2, μ_2, γ_2 of the second and the third equation in system (13) are unknown. Therefore, inspired by the works of [32], [37] and [38], we rewrite system (21) into the following affine unknown parameters observer normal form:

$$\begin{cases} \dot{\xi} = A(w, y)\xi + \varphi(y) + \phi(w, y)\theta \\ y = C\xi \end{cases} \tag{23}$$

where $\xi = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 \end{bmatrix}^T$, $\theta = \begin{bmatrix} b_2 & \frac{\mu_2}{\lambda_2} & \mu_2 & \lambda_2 \end{bmatrix}^T$

$$A(w, y) = \begin{bmatrix} 0 & 0 & 0 \\ \alpha_2(w, y) & 0 & 0 \\ 0 & \alpha_3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{I}{l(w)} & 0 & 0 \\ 0 & \lambda_2 & 0 \end{bmatrix} \tag{24}$$

$$\phi(w, y) = \begin{bmatrix} l(w) & 0 & 0 & 0 \\ 0 & \frac{\mu_1 S - b_1}{\lambda_1 S} & 0 & 0 \\ 0 & 0 & \frac{\ln(S)}{\lambda_1} & \frac{(S+I)\ln(S)+S}{\lambda_1} \end{bmatrix}$$

and

$$\varphi(y) = \begin{bmatrix} 0 \\ \left(\frac{\mu_1 S - b_1}{\lambda_1 S}\right) I - \left(b_1 - \mu_1 S - (\gamma + \mu_1) I\right)\frac{\ln(S)}{\lambda_1} \\ \frac{\mu_1}{\lambda_1} - \frac{b_1}{\lambda_1 S} \end{bmatrix}$$

Similarly to [38] and [37], it is assumed as well that there exist positive constants α, β, T such that

$$\alpha I \leq \int_t^{t+T} \Lambda^T(\tau)\, C^T \Sigma(\tau) C(\tau)\, \Lambda(\tau)\, dr \leq \beta I$$

and

$$\alpha I \leq \int_{t}^{t+T} \Psi(t,\tau)^T C^T \Sigma(\tau) C \Psi(t,\tau)\, dr \leq \beta I \qquad \forall t \geq t_0$$

where Ψ is the transition matrix for the system

$$\begin{cases} \dot{x} = A(y,w)x \\ y = Cx \end{cases}$$

and Σ is a positive definite bounded matrix. Then, based on the results in [32], [37] and [38], the following system:

$$\begin{aligned}
\dot{\widehat{x}} &= A(y,w)\widehat{x} + \varphi(y) + \phi(y,w)\widehat{\theta} \\
&\quad + \left\{\Lambda S_\theta^{-1}\Lambda^T C^T + S_x^{-1}C^T\right\}\Sigma\,(y - C\widehat{x}) && (25)\\
\dot{\widehat{\theta}} &= S_\theta^{-1}\Lambda^T C^T \Sigma (y - C\widehat{x}) && (26)\\
\dot{\Lambda} &= \left\{A(y,w) - S_x^{-1}C^T C\right\}\Lambda + \phi(y,w) && (27)\\
\dot{S}_x &= -\rho_x S_x - A(y,w)^T S_x - S_x A(y,w) + C^T \Sigma C && (28)\\
\dot{S}_\theta &= -\rho_\theta S_\theta + \Lambda^T C^T \Sigma C \Lambda, \quad S_x(0),\ S_\theta(0) > 0 && (29)
\end{aligned}$$

where ρ_x and ρ_θ are positive constants, is an exponential adaptive observer for the nonlinear system (23).

Remark 3: According to [33], as we have the form (24), then (28) can be simplified to algebraic equation (10). In this case, it implies the following equation:

$$R_\rho = \begin{pmatrix} \frac{6}{\rho_x^5} & \frac{-3}{\rho_x^4} & \frac{1}{\rho_x^3} \\ \frac{-3}{\rho_x^4} & \frac{2}{\rho_x^3} & \frac{-1}{\rho_x^2} \\ \frac{1}{\rho_x^3} & \frac{-1}{\rho_x^2} & \frac{1}{\rho_x} \end{pmatrix}$$

A. Simulation results

The figures (4)-(8) present the simulation results obtained with adaptive observer where $\rho_x = 0.77$, $\rho_\theta = 0.05$, $\Sigma = 1$ and the initial conditions are the same as before. The fast convergences show the efficiency of the used approach to estimate the states (M, V) and to identify the parameters (b_2, λ_2, μ_2).

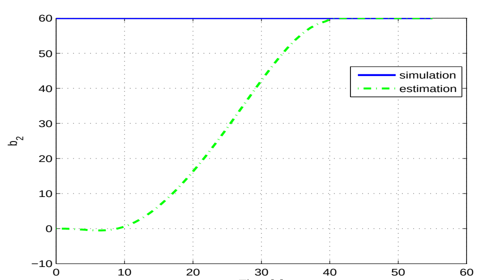

Fig. 4. Identification of parameter b_2

VI. CONCLUSIONS AND FUTURE WORKS

The observer design for simultaneous states estimation and parameter identification of dengue epidemic model is studied. The efficiency of the proposed approach mixing the change of coordinates and extended dynamics to construct extended observer normal form is illustrated by the application to practical example.

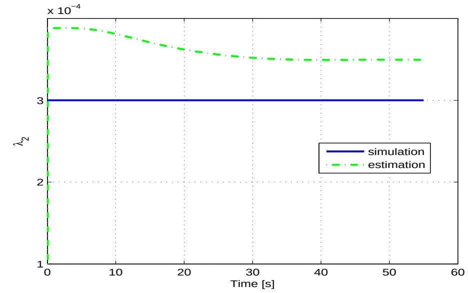

Fig. 5. Identification of parameter λ_2

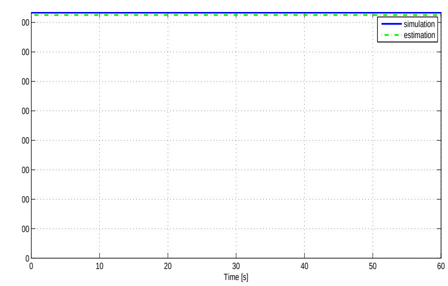

Fig. 6. Identification of parameter μ_2/λ_2

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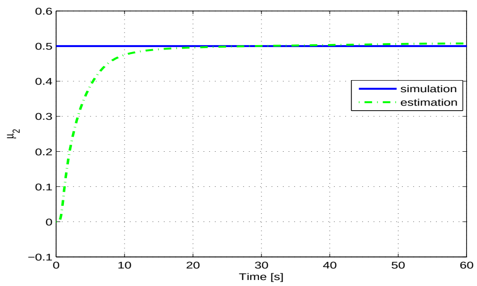

Fig. 7. Identification of parameter μ_2

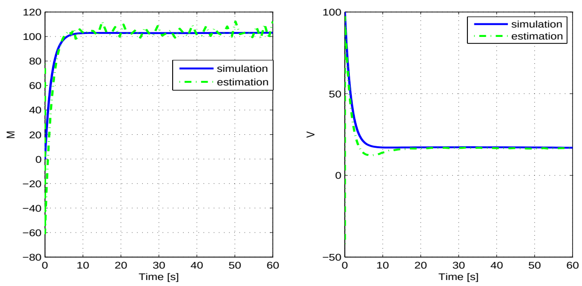

Fig. 8. Estimation of M and V

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